Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-08-30

Conceptual questions

Question 1. Find a parametrization for the curve $y^2 = x^3$ that traces out the entire curve (not just part of it!).

Question 2. Let $x = f(t), y = g(t)$ be a parametric curve such that $g'(3) = 0$. What can you conclude (if anything) about the tangent line at $t = 3$?

Question 3. True or false: for a parametric curve $x = f(t), y = g(t)$, we have $\mathrm{d}^2y/\mathrm{d}x^2 = \dfrac{\mathrm{d}^2y/\mathrm{d}t^2}{\mathrm{d}^2x/\mathrm{d}t^2}$.

Computations

Problem 1. Find a Cartesian equation for the parametric curve $x = t^3 + t, y = t^2 + 2$. Hint: compute x^2.

Find the slope of this curve at the point $(10, 6)$. If you remember implicit differentiation, try using that on the Cartesian equation and check that you get the same answer.

Problem 2 (Stewart §10.2.54)**.** Compute the arclength of the "astroid" $x = \cos^3 t, y = \sin^3 t$ depicted in Figure 1. (Stewart §10.2.34 asks you for the area.)

Problem 3. There are two points on the curve

$$x = 2t^2, y = t - t^2, -\infty < t < \infty$$

where the tangent line passes through the point $(10, -2)$. Find these two points.

CONCEPTUAL

① $x = t^2$
$y = t^3$ $\quad -\infty < t < \infty$

② If we also knew that $f'(3) \neq 0$ then we could conclude that the tangent at $t=3$ is horizontal. But as it stands, we can't conclude that.

③ Very false! See the beginning of §10.2.

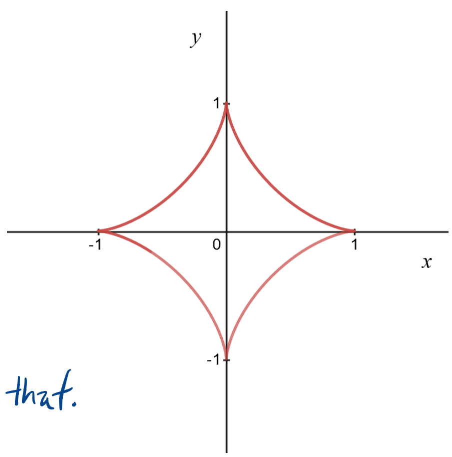

FIGURE 1. Problem 3

COMPUTATIONAL

① $x^2 - y^3 + 4y^2 - 5y + 2 = 0$

$(10,6)$ is @ $t = 2$.

$\dfrac{dy}{dx} = \dfrac{2t}{3t^2+1}$ which evaluates to $\boxed{4/13}$.

② 6

③ $(2,0)$ and $(50,-20)$.